PHYSICS NYB-10/11 Winter 2007

Lecture 7: Electric potential

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Review

- A charge q_0 moving from point A to point B in an electric field $\vec{E}$ feels a force $\vec{F}_e = q_0\vec{E}$ at every point between A and B
- This force does work on the charge when it undergoes a displacement from point A to point B

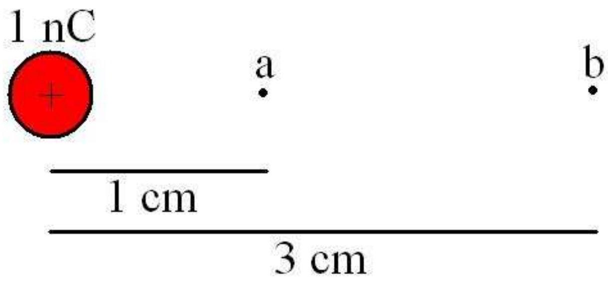
$$W_e = \int_A^B \vec{F}_e \cdot d\vec{s} = q_0 \int_A^B \vec{E} \cdot d\vec{s}$$

- Note that the work doesn't depend on the actual path taken from A to B

Review

- The work done by the field on the charge is minus the change in the potential energy
$$W_e = -\Delta U_e$$
- The total energy is conserved,
$$E_{tot} = K + U = \text{ constant}$$
- The electric potential energy for two charges separated by a distance r is
$$U(r) = k_e \frac{q_0 q}{r}$$

Review



What is the potential energy of a proton at a and b?
What is the speed at b of a proton that was moving to the right with a speed $v = 4 \times 10^5$ m/s at a?
What is the speed at a of a proton that was moving to the left with a speed $v = 4 \times 10^5$ m/s at b?

Review

First, we find the potential energy when the proton is at a and at b. We find $U = k_e \dfrac{q_0 q}{r}$ where $q_0 = 1.6 \times 10^{-19}$ C and $q = 1 \times 10^{-9}$ C. The distance r is 0.01 m and 0.03 m respectively for points a and b. This leads to $U_a = 1.44 \times 10^{-16}$ J and $U_b = 4.79 \times 10^{-17}$ J.

The final kinetic energy of a particle moving in this field can be found from applying conservation of energy $U_i + K_i = U_f + K_f$ which means $K_f = K_i + U_i - U_f$.

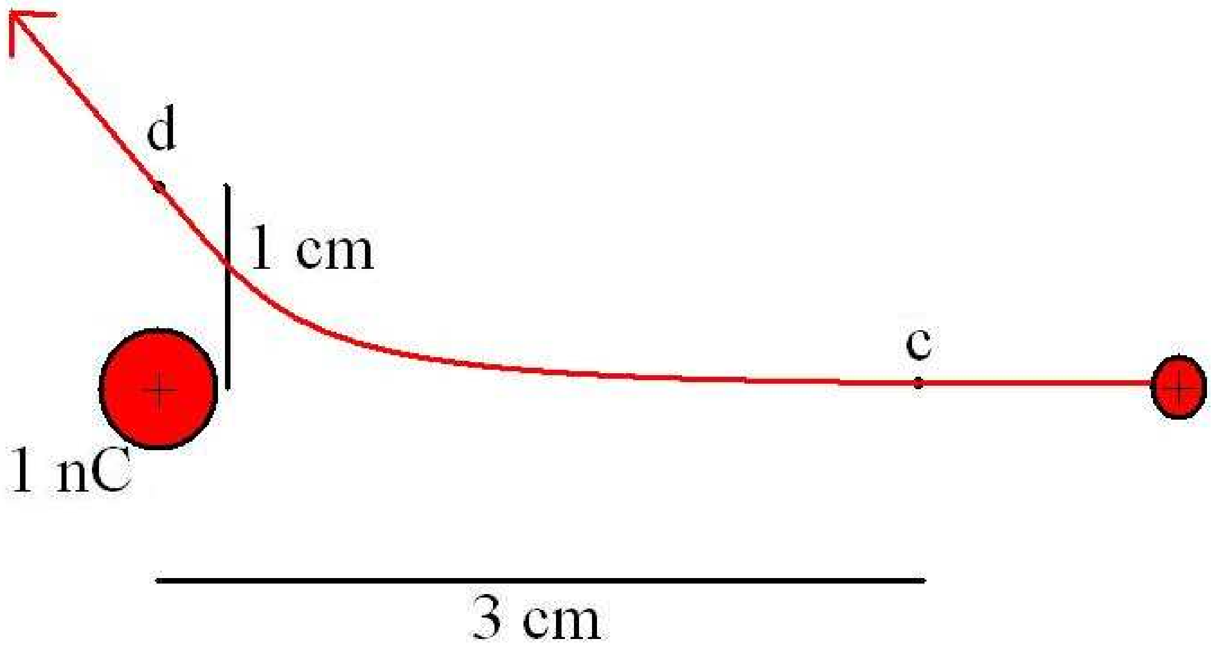
Review

This means

$$
\begin{aligned}
\frac{1}{2} m_p v_f^2 &= \frac{1}{2} m_p v_i^2 + U_i - U_f \\
v_f &= \sqrt{v_i^2 + \frac{2}{m_p}(U_i - U_f)}
\end{aligned}
$$

Plugging in values, we find that the one going from point a to point b has $v_f = 5.24 \times 10^5$ m/s, while the one going from b to a has $v_f = 2.12 \times 10^5$ m/s.

Review



What is the speed at d of a proton that was moving with a speed $v = 4 \times 10^5$ m/s at c?

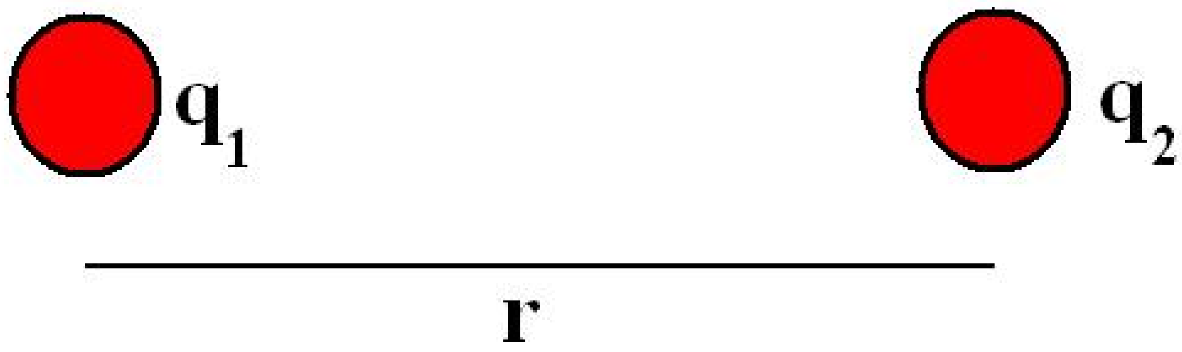
Review

Since the initial and final values of the potential are exactly the same as in the problem we did before for the proton going from point b to a, we don't need to do any calculation to know that the final speed of the proton is $v_f = 2.12 \times 10^5$ m/s.

U_e of multiple point charges

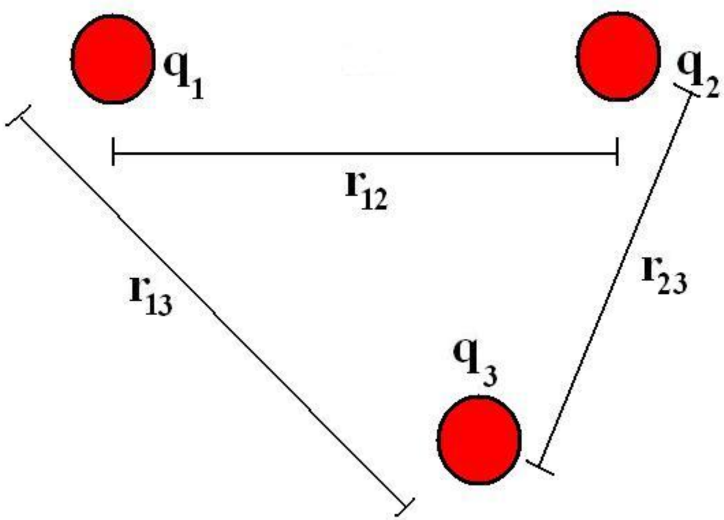
The potential energy of two point charges separated by a distance r is given by

$$U = k_e \frac{q_1 q_2}{r}$$



What happens if we bring a third charge q_3 into the picture?

U_e of multiple point charges

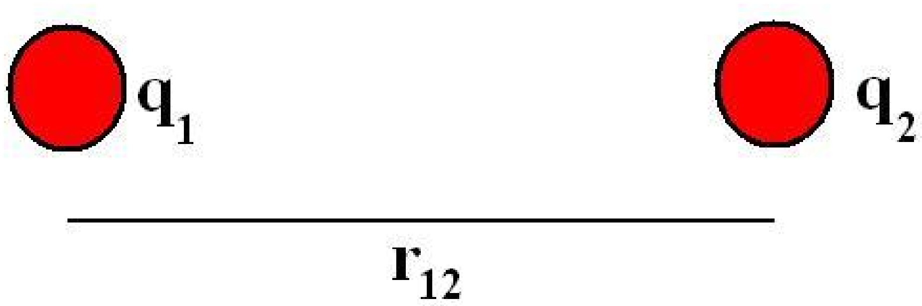


What happens if we release q_3?

Since here all charges are positive, it will accelerate away from q_1 and q_2, until it reaches infinity. The work done by charge q_1 on q_3 in that case is $k_e \dfrac{q_1 q_3}{r_{13}}$ while the work done by charge q_2 on q_3 is $k_e \dfrac{q_2 q_3}{r_{23}}$.

U_e of multiple point charges

Then we are left with



What happens if we release q_2?

It will go off to infinity, with charge q_1 doing an amount of work $k_e \frac{q_1 q_2}{r_{12}}$ on it.

U_e of multiple point charges

So the total amount of work done if we release the system is $k_e \frac{q_1 q_2}{r_{12}} + k_e \frac{q_1 q_3}{r_{13}} + k_e \frac{q_2 q_3}{r_{23}}$ which means that the total potential energy in the system originally was

$$U_e = k_e \left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right)$$

and more generally,

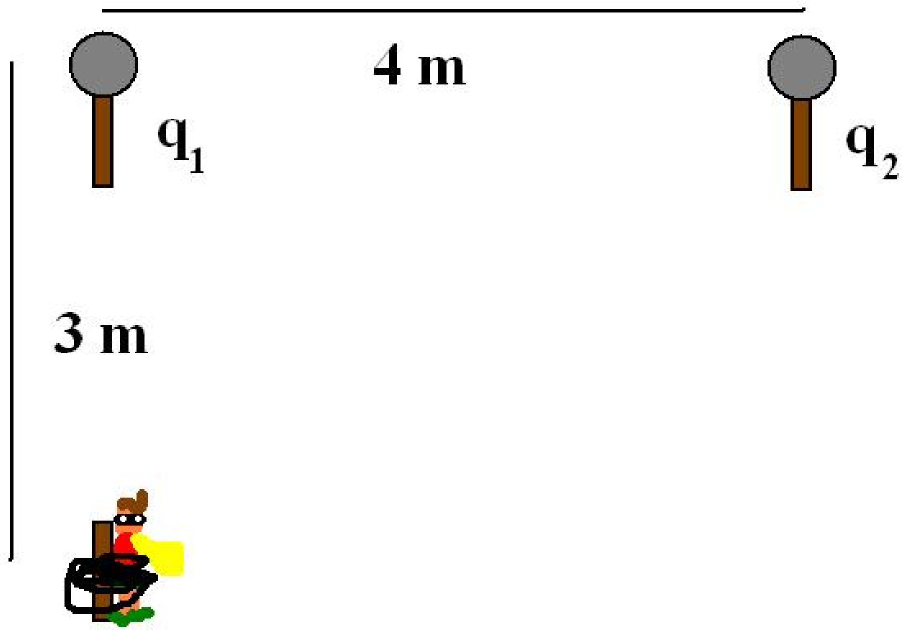
$$U_{tot} = \sum_{i=1}^{n} \sum_{j=1 (j \neq i)}^{n} \frac{1}{2} k_e \frac{q_i q_j}{r_{ij}}$$

U_e of multiple point charges

- The total potential energy is the kinetic energy the charges would eventually gain if we were to release them
- Alternatively, it is the energy that was required to bring them together in the first place
- Left on their own, charges move so as to minimize their total potential energy

Example

Robin has been captured by the Joker and is in need of rescue. He has been tied to a post as shown below, and the Joker has placed charges $q_1 = 12$ mC and $q_2 = -25$ mC on two other posts close to Robin. Batman has just taken his Batsuit out of the dryer, and the suit has a charge of $q_B = 5\ \mu$C on it. How much work will Batman need to do to come from very far away to the point where Robin is located? (Treat everything as point charges)



Example

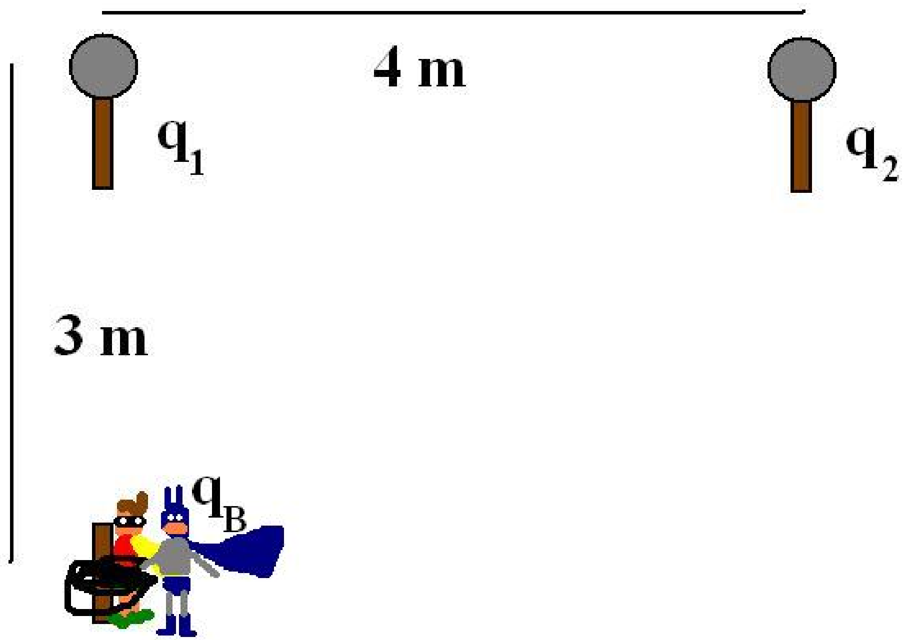
When Batman is very far away, the potential energy associated with him and the two charges is zero. When he moves to where Robin is located, the potential energy between him and the charges will be $U_e = k_e \frac{q_1 q_B}{r_1} + k_e \frac{q_2 q_B}{r_2}$. Now the change in the potential energy is minus the work done by the electric forces on Batman as he moves in. It is also the work Batman did on the system as he moved in. If we plug in numbers, we find

$$\Delta U_e = -W_e = W_B = -44.95 \text{ J}$$

Notice that this is negative. This means the system did positive work on Batman; it pulled him in! On the other hand, Batman did negative work as he came in, meaning his presence took energy out of the system. This means however that as Batman leaves, he'll have to do positive work because the system will now *hold him back*.

Example

What is the total electric potential energy of the system when Batman is standing next to Robin?



Example

We have a system of three charges, so the total potential energy of the system is

$$U_e = k_e \frac{q_1 q_2}{r_{12}} + k_e \frac{q_1 q_3}{r_{13}} + k_e \frac{q_3 q_2}{r_{32}}$$

which, plugging the values in (and noticing that we have a "345" triangle here), gives us

$$U_e = -6.74 \times 10^5 \text{ J}$$

Electric Potential

- The electric force between two point charges is $\vec{F}_e = k_e \frac{q_1 q_2}{r^2} \hat{r}$
- The reason this happens is that any charge q creates an *electric field* $\vec{E} = k_e \frac{q}{r^2} \hat{r}$
- A charge q_0 place in an electric field $\vec{E}$ feels a force $\vec{F}_e = q_0 \vec{E}$
- The potential energy of two point charges is $U_e = k_e \frac{q_1 q_2}{r}$
- Can we associate something with any charge q that leads to the potential energy?

Electric Potential

Any charge q creates an *electric potential*

$$\boxed{V(r) = k_e \frac{q}{r}}$$

When we place a charge q_0 inside an electric potential V, there is potential energy

$$\boxed{U_e = q_0 V}$$

Since $\Delta U_e = -q_0 \int_A^B \vec{E} \cdot d\vec{s}$, we can write

$$\Delta V(r) = -\int_A^B \vec{E} \cdot d\vec{s}$$

Electric Potential

- Notice that the electric potential is a *scalar!*

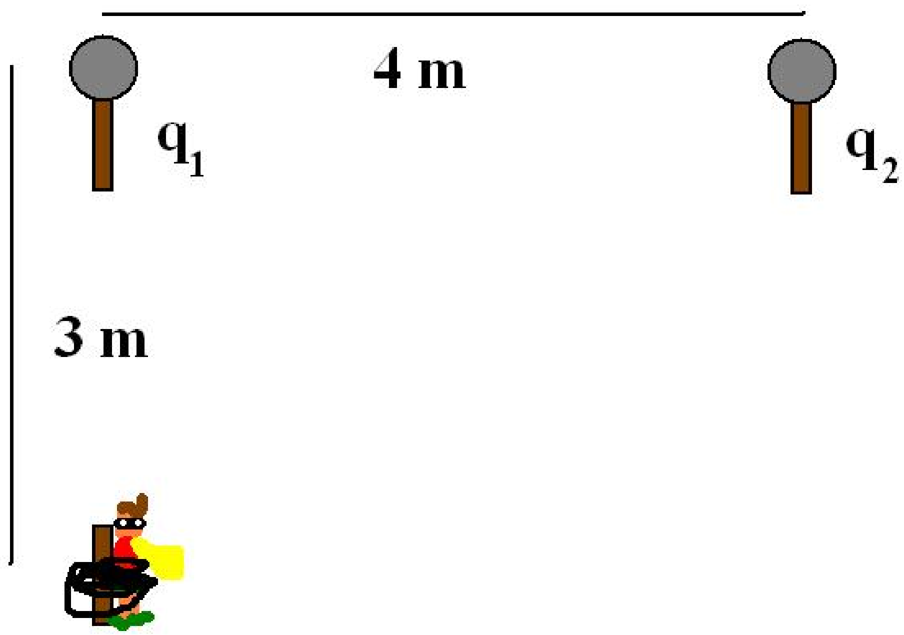
$$\boxed{V(r) = k_e \frac{q}{r}}$$

- The electric potential, V, is measured in *volts*, V...
- Yes, it can be confusing, but that's just the way it is.
- When multiple charges are present, the net electric potential is the sum of the potential created by each charge

$$\boxed{V_{tot}(r) = \sum_i k_e \frac{q_i}{r_i}}$$

Example

Robin has been captured by the Joker and is in need of rescue. He has been tied to a post as shown below, and the Joker has placed charges $q_1 = 12$ mC and $q_2 = -25$ mC on two other posts close to Robin. What is the electric potential at the point where Robin is tied? What is the electric potential far away in the BatCave? (Treat everything as point charges, and give *both the magnitude and direction* of the net potential.)



Example

First, I hope everyone realized that the part about direction is a trick question... Electric potential is a scalar!!!

Its value at Robin's location is the sum of the potential created by charges q_1 and q_2. This will be equal to

$$V_{tot} = k_e \frac{q_1}{r_1} + k_e \frac{q_2}{r_2}$$

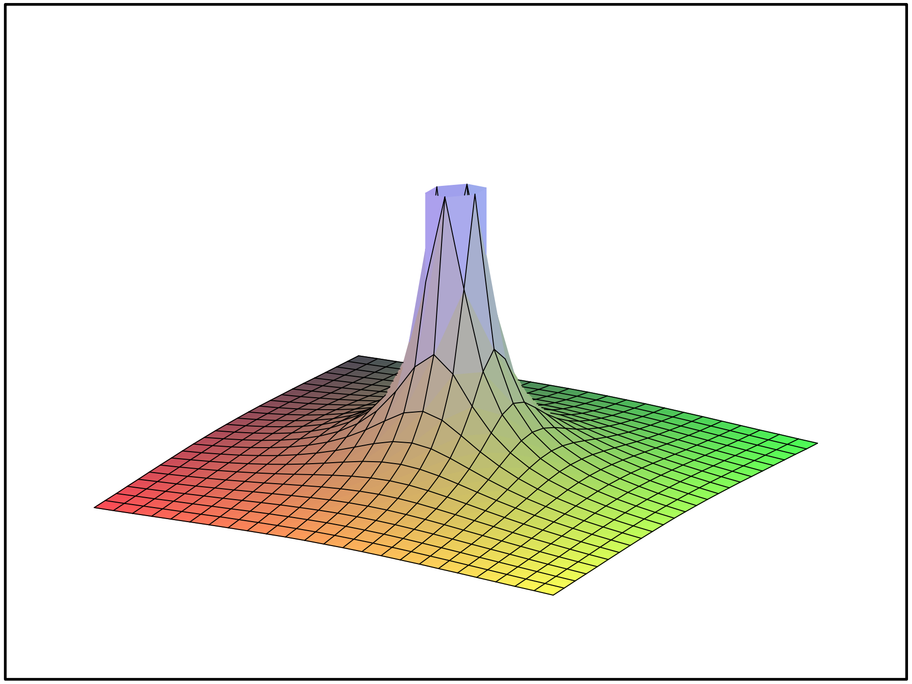
which, plugging in the data from the problem, gives us

$$V_{tot} = -8.99 \times 10^6 \text{ V}$$

As far as the potential far away in the BatCave, the distance is very large, but not enough that we can consider $V_{tot} \approx 0$. Indeed, a typical distance would be on the order of tens of kilometers, say $\approx 10^4$ m. This will lead to a potential on the order of 10^2 V.

Electric Potential vs Electric Field

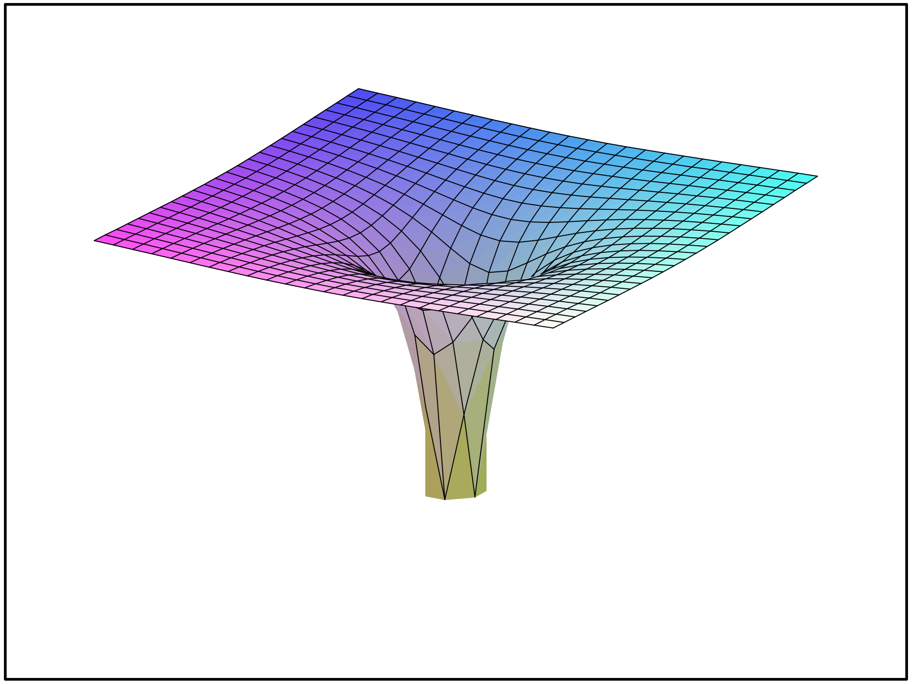
The electric potential can be represented with surfaces



- The height represents the value of the potential V, which is set to zero at infinity.
- A positive charge will feel a force making “roll” downhill.
- A negative charge will feel a force making “roll” *uphill*.

Electric Potential vs Electric Field

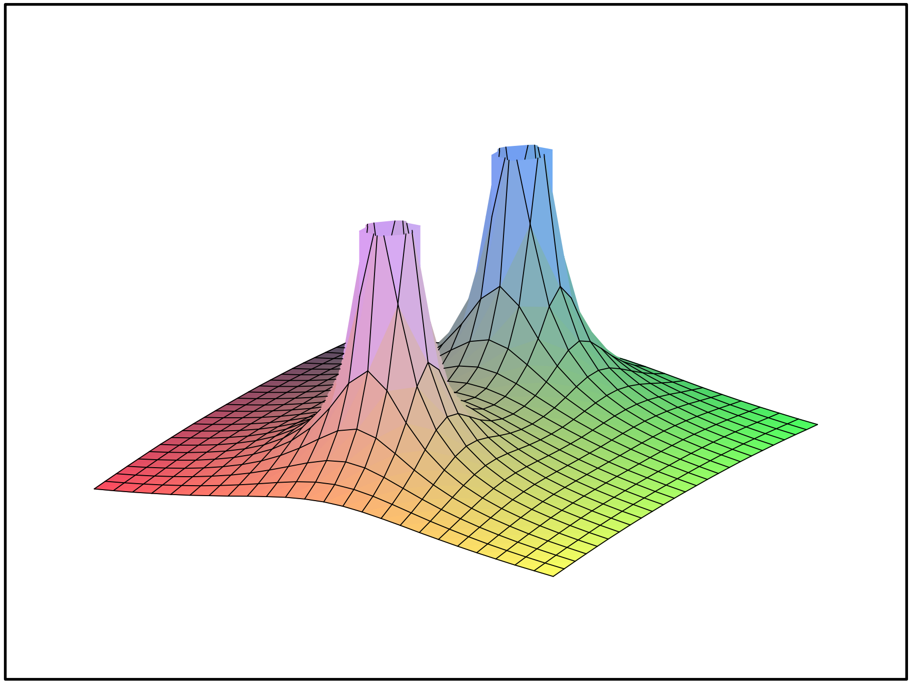
The electric potential can be represented with surfaces



- The bigger the slope, the bigger the force.
- The force is proportional to $|\vec{E}|$.

Electric Potential vs Electric Field

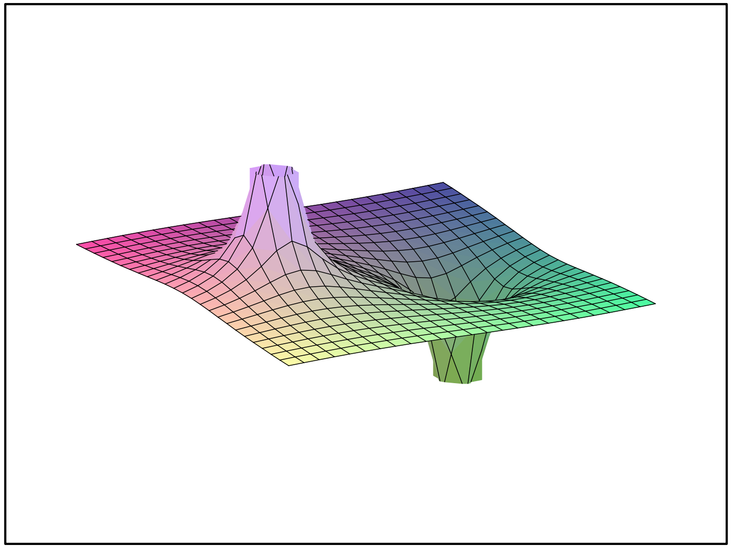
The electric potential can be represented with surfaces



- Therefore the magnitude of the electric field is proportional to the *slope* of V.
- The electric field points towards decreasing V.

Electric Potential vs Electric Field

The electric potential can be represented with surfaces



- Mathematically, $\boxed{-\frac{\partial}{\partial r}V(r)}$
- If we'd rather have the x component of $\vec{E}$, we do $\boxed{-\frac{\partial}{\partial x}V(x,y,z)}$. Same thing goes for the y or z components.

Units

So now we know that

$$\vec{E} = \frac{\vec{F}}{q_0}$$

which means that the electric field is measured in N/C. We also have that

$$\vec{E} = -\left(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right)$$

which means that the electric field is measured in V/m.

This implies that

V/m = N/C

What to read for next lecture

- 25.4, 25.6, 25.8